Random Variable

X "minimal" support S_X (minimal = default)

X = result obtained from rolling a dice $S_X = \{1,2,3,4,5,6\}$

X = " " flipping a coin $S_X = \{0,1\}$

$H \to 1$
$T \to 0$

$P[X=3]$, $P[X=2]$, $P[X=8]$

$P[X=c]$

$$p_X(x) = \begin{cases} 1/6, & x = 1,2,3,4,5,6, \\ 0, & \text{otherwise.} \end{cases}$$

$p_X(x) \equiv P[X = x]$

probability mass function

$$p_X(x) = \begin{cases} 1/6, & x \in \{1,2,3,4,5,6\}, \\ 0, & \text{otherwise.} \end{cases}$$

$X \sim \text{Bernoulli}(p)$ $$p_X(x) = \begin{cases} 1-p, & x=0, \\ p, & x=1, \\ 0, & \text{otherwise} \end{cases}$$ $S_X = \{0,1\}$

$X \sim \text{Uniform}(S)$ $$p_X(x) = \begin{cases} 1/|S|, & x \in S, \\ 0, & \text{otherwise.} \end{cases}$$

$X \sim \text{Uniform}(\underbrace{[n]}_{\{1,2,3,\dots,n\}})$ $$p_X(x) = \begin{cases} 1/n, & x \in [n], \\ 0, & \text{otherwise.} \end{cases}$$

$X \sim \text{Poisson}(\alpha)$ $$p_X(x) = \begin{cases} e^{-\alpha}\frac{\alpha^x}{x!}, & x = 0,1,2,\dots \\ 0, & \text{otherwise.} \end{cases}$$

$X \sim \text{Binomial}(n,p)$ $$p_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0,\dots,n \\ 0 & \text{otherwise.} \end{cases}$$

$X \sim \text{Geometric}_1(p)$ $$p_X(x) = \begin{cases} (1-p)^{x-1} p, & x = 1,2,3,\dots \\ 0, & \text{otherwise.} \end{cases}$$

$\text{Geometric}_0(p)$ $$p_X(x) = \begin{cases} (1-p)^{x} p & x = 0,1,2,3\dots \\ 0, & \text{otherwise.} \end{cases}$$

Consider the random variable specified in each part below

Write down (i) its (minimal) support, (ii) its pmf, and (iii) find $P[X<1]$

(a) $X \sim \text{Poisson}(\underset{\alpha}{2})$ (iv) find $P[X>1]$

(i) $S_X = \{0, 1, 2, \dots\}$

(ii) $$p_X(x) = \begin{cases} e^{-\alpha}\dfrac{\alpha^x}{x!}, & x \in S_X, \\ 0, & \text{otherwise} \end{cases} = \begin{cases} e^{-2}\dfrac{2^x}{x!}, & x \in S_X, \\ 0, & \text{otherwise.} \end{cases}$$

(iii) $$P[X<1] = p_X(0) = e^{-2}\frac{2^0}{0!} = e^{-2} \approx 0.135$$

Only "0" satisfies the condition "<1"

(iv) $$P[X>1] = p_X(2) + p_X(3) + p_X(4) + \dots$$

"2", "3", "4", ... satisfy the condition ">1"

$$= 1 - P[\underbrace{X \leq 1}_{\text{opposite case}}]$$

$$= 1 - \underbrace{\left(p_X(0) + p_X(1)\right)} = 1 - e^{-2} - e^{-2}\frac{2^1}{1!} = 1 - e^{-2} - 2e^{-2}$$

≈ 0.594

Only "0" and "1" satisfy the condition "≤1"

(b) $X \sim \text{binomial}(\underset{n}{1}, \underset{p}{1/2})$

(i) $S_X = \{0, 1, \dots, n\} = \{0, 1\}$

(ii) $$p_X(x) = \begin{cases} 1/2, & x = 0, \\ 1/2, & x = 1, \\ 0, & \text{otherwise} \end{cases}$$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \binom{1}{x} \underbrace{\left(\tfrac{1}{2}\right)^{\cancel{x}} \left(\tfrac{1}{2}\right)^{\cancel{1-x}}}$$

$$= \binom{1}{x}\frac{1}{2}$$

(Remark: X is also Bernoulli$(\frac{1}{2})$ and uniform$(\{0,1\})$)